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MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

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Question	Answer	Marks	Guidance
1	$m = {}^{-6}/_{6} = -1$	**M1**	Attempt gradient
	$y - 5 = -(x - 2)$	**M1**	Attempt equation of line
	$y + x = 7$	**A1**	Obtain correct equation aef but must be simplified to three terms
	$9 + -2 = 7$	**A1**	Justify that (–2, 9) is on line
2(a)(i)	$\Delta = b^2 - 4ac$	**M1**	Attempt discriminant
	$= 9 - 20 = -11$	**A1**	Obtain – 11
2(a)(ii)	No real roots	**B1***	**FT** Correct conclusion, following *their* numerical discriminant – allow BOD if *their* (i) had square root present
	as $-11 < 0$	**B1d***	**FT** Correct reasoning, using discriminant (insufficient to just state that roots are imaginary as the reason)
2(b)	$\Delta = 9 - 20k = 0$	**M1**	Equate attempt at discriminant to 0 Allow M1 if using an incorrect discriminant formula if this is the same as used in (a)(i)
	$k = \dfrac{9}{20}$	**A1**	Obtain ${}^{9}/_{20}$ oe Allow BOD for both M1 and A1 if equating the square root of the discriminant to 0
3	$\theta = \tan^{-1}0.1 - 10^{\circ}$	**M1**	Attempt θ using correct order of operations
	Obtain at least one correct value	**A1**	inc – 4.29
	Attempt at least one value of θ in range	**M1**	allow incorrect principal angle + 180º
	$\theta = (-4.29^{\circ}), 175.7^{\circ}, 355.7^{\circ}$	**A1**	Obtain both angles, and no others in range If using $\tan(A + B)$ approach: **B1** for correct identity **B1** for correct expression for $\tan\theta$ **M1** for attempting θ (in range) from $\tan\theta = k$ **A1** for both angles, and no others in range
4(i)	$u_2 = i(1 + i)$, $u_3 = i(-1 + i)$ or $i(i + i^2)$ oe	**M1**	Attempt correct process to find at least u_2 and u_3
	$u_2 = -1 + i$, $u_3 = -1 - i$,	**A1**	Correct, simplified, u_2 and u_3
	$u_4 = 1 - i$, $u_5 = 1 + i$, $u_6 = -1 + i$	**A1**	Fully correct and simplified
4(ii)	Periodic (with period 4)	**B1**	Any equivalent description Allow geometric

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4(iii)	Every four terms sum to zero, so $S_{72} = 0$	**M1**	Attempt to use repeating pattern
	hence sum is 1 + i	**A1**	Obtain 1 + i (NB $u_{73} = 1 + \mathrm{i}$, but this is M0)
	OR $S_{73} = \dfrac{(1+\mathrm{i})(1-\mathrm{i}^{73})}{1-\mathrm{i}}$	**M1**	Attempt sum of GP with $r = \mathrm{i}$
	= 1 + i	**A1**	Obtain 1 + i
5(i)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{1+x^2} = \dfrac{x}{\sqrt{1+x^2}}$	**M1**	Attempt use of chain rule to obtain $kx(1+x^2)^{-\frac{1}{2}}$
		A1	Obtain correct derivative, soi
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{x}{\sqrt{1+x^2}} = \dfrac{\sqrt{1+x^2} - \dfrac{x^2}{\sqrt{1+x^2}}}{1+x^2}$	**M1**	Attempt use of quotient rule (allow uv' – u'v in numerator)
		A1	Obtain correct numerator or denominator – must now be from correct rule
		A1	Obtain correct derivative aef
	OR $\dfrac{\mathrm{d}}{\mathrm{d}x}(1+x^2)^{-\frac{1}{2}} = -x(1+x^2)^{-\frac{3}{2}}$	**M1**	Attempt use of chain rule to obtain $kx(1+x^2)^{-\frac{3}{2}}$
		A1	Obtain correct derivative, soi
		M1	Attempt use of product rule
	$\dfrac{\mathrm{d}}{\mathrm{d}x}x(1+x^2)^{-\frac{1}{2}} = (1+x^2)^{-\frac{1}{2}} - x^2(1+x^2)^{-\frac{3}{2}}$	**A1**	Obtain one correct term – from correct rule
		A1	Obtain correct derivative aef
5(ii)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{x}{\sqrt{1+x^2}} = \dfrac{1}{\left(1+x^2\right)^{\frac{3}{2}}}$	**B1***	Simplify to correct useable form (may be seen in part (i))
	$1+x^2 > 0$ so it is increasing	**B1d***	Conclude appropriately – must refer to both positive gradient (could be algebraic) and increasing

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6	$\int y^2 \mathrm{d}y = \int \frac{x+1}{x} \mathrm{d}x$	**M1***	Separate variables
	$= \int 1 + \frac{1}{x} \mathrm{d}x$	**M1**	Attempt to deal with improper fraction (could include integration by parts)
		A1	Correct useable expression
	$\frac{1}{3}y^3 = \ldots$	**A1**	Correct LHS
	$x + \ln\lvert x\rvert + c$	**A1**	Correct RHS
	$9 = 1 + \ln 1 + c$	**M1d***	Substitute $x = 1$, $y = 3$ to find c
	$y = \sqrt[3]{3(x + \ln\lvert x\rvert + 8)}$	**A1**	Obtain correct equation, in required form Allow ln$x$ without modulus sign
7(i)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -2\sin\theta,\ \frac{\mathrm{d}y}{\mathrm{d}\theta} = 3\cos\theta$	**B1**	Both derivatives correct
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}} \quad = \frac{3\cos\theta}{-2\sin\theta}$	**M1**	Attempt at parametric differentiation soi
	$= -\frac{3}{2}\cot\theta$	**A1**	Obtain correct unsimplified derivative and then simplify to given answer
7(ii)	$y - 3\sin\theta = -\frac{3}{2}\cot\theta(x - 2\cos\theta)$	**M1***	Attempt equation of tangent, in terms of θ
		A1	Obtain correct equation aef – could be implied by correct c if using $y = mx + c$
	$0 - 3\sin\theta = -\frac{3}{2}\cot\theta(x - 2\cos\theta)$	**M1d***	Attempt x-intercept – substitute $y = 0$ to get a value for x
	$x = 2\sec\theta$	**A1**	Obtain $x = 2\sec\theta$, with sufficient detail seen
	$y - 3\sin\theta = -\frac{3}{2}\cot\theta(0 - 2\cos\theta)$	**M1d***	Attempt y-intercept – substitute $x = 0$ to get a value for y
	$y = 3\operatorname{cosec}\theta$	**A1**	Obtain $y = 3\operatorname{cosec}\theta$, with sufficient detail seen
	midpoint is $\left(\frac{1}{2}\times 2\sec\theta, \frac{1}{2}\times 3\operatorname{cosec}\theta\right)$ $= \left(\sec\theta, \frac{3}{2}\operatorname{cosec}\theta\right)$	**A1**	Show given answer for midpoint – must show some working so A0 if straight from intercepts to given answer
7(iii)	$\frac{4}{\sec^2\theta} + \frac{9}{\left(\frac{3}{2}\operatorname{cosec}\theta\right)^2}$ $= 4\cos^2\theta + 4\sin^2\theta = 4$	**M1**	Substitute coords from (ii)
		A1	Convincingly show that midpoint is on curve

Question	Answer	Marks	Guidance
8(i)	$(Ax + B)(x - 2) + C(x^2 + 1) = 7x^2 - 12x + 1$	**M1**	Set up correct identity
	$A = 6$	**A1**	
	$B = 0$	**A1**	
	$C = 1$	**A1**	
8(ii)	$\int \frac{7x^2 - 12x + 1}{(x^2 + 1)(x - 2)} dx = \int \frac{6x}{x^2 + 1} dx + \int \frac{1}{x - 2} dx$	**M1**	Integrate first fraction to $k\ln(x^2 + 1)$
	$= 3\ln\lvert x^2 + 1\rvert$	**A1 FT**	Obtain correct integral, following *their A*
	$+\ln\lvert x - 2\rvert$	**B1 FT**	Obtain correct integral of second fraction, following their C (allow brackets soi not modulus each time)
		M1*	Attempt correct use of limits in any changed function – could be just one of the two terms
	(3ln2 + ln1) – (3ln1 + ln2)	**B1d***	Use or imply that $\ln\lvert -k\rvert = \ln k$ B0 if using log laws with negative numbers
	= ln4	**A1**	Obtain ln4, or 2ln2 (can follow B0) If *their B* in (i) was non-zero then the first term will need to be split into two fractions so that one of *their* fractions is of the correct form for M1A1. Condone the third term being present (correct or incorrect) for the first 5 marks.

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9(i)	**Substitution** $\int (u+2)u^{\frac{3}{2}}du$	**M1*** **A1**	Substitute $u = x - 2$ Correct integrand, in terms of u
	$= \int u^{\frac{5}{2}} + 2u^{\frac{3}{2}}du$	**M1d***	Expand brackets and attempt integration
	$= \frac{2}{7}u^{\frac{7}{2}} + \frac{4}{5}u^{\frac{5}{2}} + c$	**A1**	Obtain correct integral (in terms of u or x) as long as consistent
	$= \frac{2}{35}u^{\frac{5}{2}}\left(5u+14\right)+c$	**M1**	Attempt to use algebraic highest common factor on expression of form $au^{\frac{7}{2}} + bu^{\frac{5}{2}}$
	$= \frac{2}{35}(5x+4)(x-2)^{\frac{5}{2}} + c$	**A1**	Obtain correct integral **AG** Condone no + c
	OR **By parts** $\int x(x-2)^{\frac{3}{2}}dx = \frac{2}{5}x(x-2)^{\frac{5}{2}} - \int \frac{2}{5}(x-2)^{\frac{5}{2}}\mathrm{d}x$	**M1***	Attempt integration by parts
		A1	Obtain correct expression
		M1d*	Attempt integration
	$= \frac{2}{5}x(x-2)^{\frac{5}{2}} - \frac{4}{35}(x-2)^{\frac{7}{2}} + c$	**A1**	Obtain correct integral
	$= \frac{2}{35}(x-2)^{\frac{5}{2}}(7x - 2(x-2)) + c$	**M1**	Attempt to use algebraic highest common factor on expression of form $ax(x-2)^{\frac{5}{2}} - b(x-2)^{\frac{7}{2}}$
	$= \frac{2}{35}(x-2)^{\frac{5}{2}}(5x+4)+c$	**A1**	Obtain correct integral **AG** Condone no + c
	OR **Using differentiation** $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2}{35}(x-2)^{\frac{5}{2}}(5x+4)+c\right)$ $= \frac{5}{2}\times\frac{2}{35}(x-2)^{\frac{3}{2}}(5x+4) + \frac{2}{35}(x-2)^{\frac{5}{2}}\times 5$	**M1***	Attempt use of product rule
		A1	Obtain one correct term
	$= \frac{1}{7}(x-2)^{\frac{3}{2}}(5x+4) + \frac{2}{7}(x-2)^{\frac{5}{2}}$	**A1**	Obtain fully correct derivative
	$= \frac{1}{7}(x-2)^{\frac{3}{2}}(5x+4+2(x-2))$	**M1d***	Attempt to use algebraic highest common factor
	$= \frac{1}{7}(x-2)^{\frac{3}{2}}(7x)$	**A1**	Obtain correct unsimplified expression
	$= x(x-2)^{\frac{3}{2}}$	**A1**	Obtain correct expression **AG**

Question	Answer	Marks	Guidance
9(ii)	$\frac{dy}{dx} = x(x-2)^{\frac{3}{2}} - x^2 + 2x$	M1 *	Attempt differentiation, using part (i)
	$x(x-2)\left(\sqrt{x-2}-1\right)=0$	A1	Obtain correct derivative
		M1d*	Equate to zero and attempt to solve, as far as non-zero value for x – allow inspection
	$x = 0$ not valid	B1	Obtain $x = 0$ and deduce no solution oe e.g. y not real, but B0 if imaginary coord given
	$x = 2, y = \frac{4}{3}$	A1	Obtain $x = 2, y = \frac{4}{3}$
	$x = 3, y = \frac{38}{35}$	A1	Obtain $x = 3, y = \frac{38}{35}$ (allow 1.09 or better) SR Allow B1 for $x = 2$ and 3 but no, or incorrect, y-values
10(i)	$a_1 = g_1 = a$	M1	Attempt at least one equation linking a, d, r
	$a + d = ar$ $a + 4d = ar^2$	A1	Obtain two correct equations
	$a + 4(ar - a) = ar^2$ $ar^2 - 4ar + a = 0$	M1	Eliminate d from equations
	$a(r^2 - 4r + 3) = 0$ $a(r - 1)(r - 3) = 0$	M1	Attempt value for r
	$a = 0,\ r = 1,\ r = 3$	A1	Obtain $r = 3$ (ignore second solution if given)
	$a_1 \neq a_2$ so $r = 3$	B1	Justify $r = 3$ as only valid solution; could be stating that $r = 1$ or $d = 0$ does not give a valid solution
10(ii)	$d = 2a_1$	B1	Correct expression for d (B0 if a not a_1)
10(iii)(a)	Geometric 5, 15, 45	B1	Correct three terms for geometric sequence
	Arithmetic 5, 15, 25	B1	Correct three terms for arithmetic sequence

Question	Answer	Marks	Guidance
10(iii)(b)	e.g. The product of 3 and an integer ending …5 ends with …5, so the terms of the geometric sequence all end …5	**M1**	Consider terms of geometric sequence Could be worded argument, or could justify with $5 \times 3^{n-1}$ (allow 5×3^n as general term)
	The arithmetic sequence covers *all* odd multiples of 5	**M1**	Consider terms of arithmetic sequence No further evidence required, but must make it clear that *all* terms are contained in AP
	So the terms of the geometric sequence are all in the arithmetic sequence.	**A1**	Conclude appropriately Alternative approach is to consider nth terms, setting up GP as $5 \times 3^{n-1}$ and AP as $5(2n - 1)$, and then comparing 3^{n-1} and $2n - 1$ **M1** justify 3^{n-1} as always odd **M1** $2n - 1$ as *all* odd numbers **A1** hence terms of GP are all in the AP